PAPER P-1181

ON THE VECTOR-I COMBAT MODEL

Alan F. Karr

August 1976

INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION

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This paper is a summary, review, and criticism of the Vector-I combat simulation model. Geographical, structural and organizational aspects of the model are treated, but the main emphasis is on consideration of attrition equations in terms of underlying sets of assumptions. Throughout the purpose of the paper is to enlarge the grounds for reasoned discussion and comparison of different combat models.

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CONTENTS

1. INTRODUCTION

This paper is a description of and commentary on the Vector-I combat model. It seeks to be fair, but constructively critical. The personal tastes and biases of the author are reflected in the emphasis on attrition computations and on underlying mathematical assumptions. The philosophical approach is that a combat model should be evaluated in terms of the perceived validity of the assumptions underlying it. This, we realize, is a difficult task when compared with methods such as empirical comparison of results with combat data, the method favored by the developers of Vector-I. Evaluation of a model in terms of assumptions means first that the assumptions be formulated and second that the model be rigorously derived from them; neither of these tasks is easy. Nevertheless, the criticisms and praise presented here are in terms of underlying assumptions, especially with regard to attrition calculations.

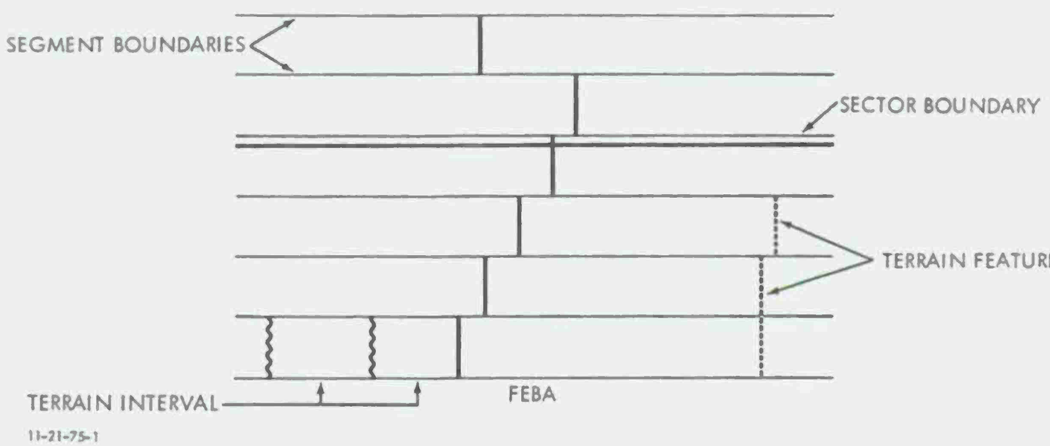
Vector-I is a computerized, iterative, deterministic simulation of mid-intensity, theater-level, ground-air combat. The report [4] on which this paper is based states that the model is intended "to provide information useful in making net assessments and general purpose force tradeoff analyses." The main characteristics of combat the model purports to represent are terrain, firepower, organization, supply consumption, movement, and activity assignment. Not all of these are, we feel, modeled equally successfully. In general, the greatest attention appears to have been devoted to the model of "assault on a hasty defense." Indeed, on the basis of the genesis of this model one is tempted to conclude that a theater-level structure has been appended to a battalion-level combat model (the Individual Unit Action model) in a not entirely careful manner. There is not a consistent level of assumptions throughout the model: those concerning ground combat are much more detailed, and (evidently) less restrictive than the others. This is particularly true for attrition computations, but is true also for representation of terrain, movement, and organization. Perhaps the model might be viewed not as an overall theater-level model, but as a detailed model of ground combat within a larger context, with the theater-level superstructure maintaining external parameters at roughly correct values. But even in this interpretation the results of the model and its ability to discern the effect of minor variations in the plethora of detailed inputs to the ground combat portion of the model should be viewed with caution. The overall effect of disaggregation of one part of the model, within the context of the assumptions made in the other parts, is unclear. Without further evidence--because of the inconsistencies so introduced--there is no reason to believe this disaggregation is of more than limited value. Essentially identical results might be obtainable with some simple model.

An interesting and positive feature of the model is the inclusion of "tactical decision rules," discussed in more detail in Section 6, by means of which the user can define a number of decision variables as functions of state variables. Any function which can be programmed into the computer is acceptable, giving the user great flexibility in modeling behavioral and organizational aspects of combat which, despite their obvious importance, are neglected in most other models. Whether any effort has been devoted to the development of large numbers of realistic rules is, of course, quite another matter. But at least the potential flexibility is impressive.

For a different treatment of several of the problems discussed here we refer the reader to descriptions [1,3] of the IDAGAM I model.

2. GEOGRAPHY AND MOVEMENT

Vector-I attempts to be more explicit and detailed about geography than some other models and succeeds to a certain extent. The geographical representation is basically a conventional one, as indicated below.



The territory assigned to each side is divided into eight sectors, each of which contains a battalion task force (or comparable unit). Sector boundaries must be the same on both sides of the piecewise linear FEBA which separates the two opposing forces. Parallel to the FEBA each sector may be divided into terrain intervals representing up to twenty-five types of terrain (five levels of visibility and five levels of trafficability). The effect of these different types of terrain is on movement rates and inputs to the calculation of attrition rate coefficients. Every sector is subdivided into segments (the same on both sides), each of which contains one battalion and is conceived as being 2 to 8 kilometers wide. All ground combat effects are computed on a per-segment basis; in this respect there is no interaction among segments. Rear regions also exist.

Also parallel to the FEBA and of sector width or wider are terrain features, namely, rivers, urban areas, and one user-defined terrain feature. While these features provide, in a strict sense, more detail within the model, their overall effect on the results is minimal and appears essentially only through effects on force movement. Attrition incurred in interactions at such features is stated in the report to be so slight as to justify being modeled solely by user-input tables.

Force movements are determined by force availability and tactical decision rules (see Section 6). Movement of the FEBA is computed from a decision to move (a decision variable computed using one of the tactical decision rules) and movement rates supplied as inputs. The amount of movement may also depend on decisions to disengage and on the type of terrain. Smoothing of the FEBA is accomplished by certain of the tactical decision rules. The relation of FEBA movement to casualty levels or relative force strengths is unclear.

Local geography is represented implicitly through dependence of attrition rates on movement, terrain, and visibility; we refer to Section 5 for further details.

3. RESOURCES AND RESOURCE ALLOCATION

Vector-I models the following resources:

A. Maneuver force weapon systems
   - 2 tank systems
   - 3 antitank weapons
   - Infantry on foot with rifles
   - Infantry mounted on armored personnel carriers
   - Infantry with machine guns
   - Infantry with area fire weapons
   - Minefields

B. Artillery forces (1 weapon class)

C. Attack helicopters (1 weapon class)

D. Air defense artillery
   - Long range
   - Short range

E. Tactical aircraft
   - 7 aircraft types
   - Shelters

Provision is also included for personnel not in maneuver units (e.g., those in rear areas). In general, weapons systems are counted explicitly but personnel are not. Each weapons system has associated with it (for purposes of counting force strengths) a certain number of personnel some, but not necessarily all, of whom are killed if the weapons system is destroyed by the enemy. So far as we can determine this is the only way personnel casualties can occur in Vector-I.

The following supplies can exist at the segment, sector, and theater levels (with some restrictions):

Ammunition by weapon type

Mines

10 types of aircraft ordnance

Aviation gasoline and related supplies

1 other class of supplies

Supply transfers and allocations are effected by the tactical decision rules; consumption of supplies is a linear function of resource usage. Supply shortages degrade force performance only by affecting activity choice (again using tactical decision rules), not by changing the values of effectiveness parameters or numbers of fully effective resources.

Sector- and theater-level reserve forces are modeled; replacements on both individual and unit bases are permitted. Assignment of replacements is also governed by tactical decision rules; see Section 6.

4. INTERACTIONS

The following is a list of the combat processes modeled in Vector-I.

A. Ground combat between maneuver units

- Assault on a hasty defense
- Advance by one side/delay by the other
- Pursuit by one side/withdrawal by the other
- Relative inaction
- Crossing of urban area (or river, or other terrain feature)
- Bypassing of urban area (or other terrain feature)

B. Artillery roles

- Counterbattery fire
- Maneuver unit support
- Deep support

C. Tactical aircraft missions

- Airbase attack
- Combat air support
- Air defense suppression
- Interdiction
- Escort
- Air defense
- 1 user-defined rear area mission

As previously noted, the only ground combat activity not modeled by means of user-input tables is that of assault on a hasty defense; the methodology used to model this activity is described and commented upon at length in Section 5 below, as are the attrition calculations for other interactions.

Air defense artillery function only in that role. Attack helicopters can be used only in support of maneuver units at the FEBA.

5. ATTRITION PROCESSES

Ground-to-Ground Attrition

The main attrition computation, for battalion-level engagements at the FEBA, is based on a heterogeneous attrition equation of the form

$$(1) \qquad \Delta n_i = \left(\sum_j A_{ij}(n)m_j\right)\Delta t \quad ,$$

where

n_i = number of target weapons of type i,
m_j = number of shooting weapons of type j,
Δt = time interval during which attrition occurs,

and where the A_{ij} are rates of attrition per unit time.

The attrition coefficients $A_{ij}(n)$ depend in a complicated and sophisticated manner on the structure of the entire target force and on factors such as range, terrain, physical characteristics of weapons systems, and posture. It is tempting to call (1) a heterogeneous Lanchester equation, but to do so is probably a misnomer. This is because the attrition coefficients are never of the classical Lanchester-square form

$$A_{ij}(n) = c_{ij}$$

for some constants c_{ij}, nor ever of the modified Lanchester-square form of [5], namely

$$A_{ij}(n) = c_j \frac{n_i}{\sum_\ell n_\ell} \quad ,$$

nor ever of Lanchester-linear form

$$A_{ij}(n) = d_{ij}n_i \ .$$

The principal distinction between weapon types in Vector-I is in terms of serial target acquisition as opposed to parallel target acquisition. In the former case a weapon searches for targets, but once a line-of-sight is acquired and an engagement begun the weapon continues the engagement, making no other acquisitions, until the engagement is terminated by destruction of the target or loss of the line of sight. Target priorities are represented by search cut-off times: the shooting weapon searches for a prescribed time for first-priority targets and if none are found, switches to searching for second-priority targets, and so on. Weapons with parallel acquisition search continuously for targets and break off an engagement whenever a higher priority target is acquired.

Of the assumptions underlying (1) the most important is that all shooting weapons operate independently of one another. This is why (1) is a linear equation in the numbers m_j of shooting weapons. Virtually every attrition model makes such an assumption (indeed, this assumption underlies each of the stochastic Lanchester models presented in [5] and [7], as well as the binomial attrition processes of [6]). The assumption can be defended on several grounds. First, one can argue that the assumption is satisfied in certain combat situations, at least to the extent that resultant errors in a simulation model are of the same order as those arising from other assumptions. In a strict sense, of course, one cannot measure the degree to which an assumption is satisfied: either the assumption holds or it does not. But in a situation so complicated as combat, such an error seems quite acceptable. If the numbers of shooting weapons are large then each does in fact operate essentially independently of most of the others, even though none operates independently of all the others. If necessary, and if certain weapons acted together in prescribed fashions one could re-define, for the purposes of attrition calculations, the weapon types, and proceed to use (1). The independence assumption can also be defended on the grounds that no alternative is known that leads to a tractable attrition equation. Hence, we believe there is a plausible argument for the basic form of (1). Unfortunately, the report [4] does not seem to contain this argument; it would be more convincing if it did.

The remaining assumptions underlying (1) are "micro" assumptions used to compute the attrition coefficients from parameters such as range, lengths of visible and invisible periods, mean times to kill given a continuous line of sight, search cut-off times, and target priorities. These assumptions are both detailed and sophisticated. For example, the state of each shooting weapon is modeled as a semi-Markov process, and limit theorems for such processes are invoked in order to calculate the attrition coefficients. One might disagree with using limiting arguments to model phenomena which are manifestly transient, but the error so introduced is probably not unduly large. However, the independence assumption previously discussed limits applicability of (1) to short times, which are precisely those to which the limiting argument is least applicable.

It is worthwhile to place the attrition process on which (1) is based in the context of the stochastic attrition processes derived in [5] and [7]. Before doing so, however, we wish to note that the interpretation as arising from an underlying Markov attrition process is not the only possible interpretation of (1). Let us suppose, momentarily, that the functions $A_{ij}(\cdot)$, and the corresponding attrition functions $A_{ji}(\cdot)$ for the other side are exogenously specified. One might then consider a <u>deterministic</u> vector-valued combat process $t \to (m(t),n(t))$, where $m(t)$ is the state of side 1 at time t and $n(t)$ that of side 2, which satisfies the differential equations

$$(2)\qquad \begin{aligned} n_i' &= -\sum_j A_{ij}(n)m_j \\ m_j' &= -\sum_i A_{ji}(m)n_i \ , \end{aligned}$$

which are certainly Lanchesterian in spirit, if not in precise form. In this case the right-hand side of (1) is indeed an approximation to $n_i(t) - n_i(t+\Delta t)$ for small values of Δt. The derivation of the functions A_{ij} and A_{ji} from "micro" stochastic hypotheses then should not be taken to imply the existence of a stochastic model of the entire attrition process, but only as an indication of the care used in deriving the coefficients of a differential, deterministic model of combat. These functions must, of course, be sufficiently smooth to ensure the existence of a unique solution to (2).

On the other hand, there exist (in general) many regular Markov attrition processes $((N_t,M_t))_{t\geq 0}$ with the interpretation that

N_t = random vector of surviving weapons on side 1 at time t,
M_t = random vector of surviving weapons on side 2 at time t,

such that

$$E[N_t(i) - N_{t+\Delta t}(i)|(N_t,M_t)] \sim [\sum_j A_{ij}(N_t)M_t(j)]\Delta t$$

and

$$E[M_t(j) - M_{t+\Delta t}(j)|(N_t,M_t)] \sim [\sum_i A_{ji}(M_t)N_t(i)]\Delta t$$

for all i,j and t and small values of Δt. The nonuniqueness arises because the stochastic interpretation of $A_{ij}(N_t)\Delta t$, for example, is that of the expected number of type i weapons on side 1 destroyed by a <u>single</u> type j weapon on side 2 within a short period $(t,t+\Delta t]$, given the state N_t of side 1 at time t. As is clear from [7], many processes can lead to the same A_{ij} and A_{ji}. In this interpretation the A_{ij} and A_{ji} must be regarded as deterministic and not as random functions arising from assumptions made in addition to those engendering the Markov attrition process. As in the deterministic case, the derivation of the attrition rate functions from detailed probabilistic models should be viewed only as an argument in favor of that particular set of attrition rate functions.

Although the report [4] does not say so explicitly, we believe that the developers of Vector-I favor the deterministic interpretation. However, either interpretation is valid provided that the attrition rate functions be interpreted in the manner described above; both interpretations are useful. We emphasize, once more, that both require the global independence assumption noted above.

Particularly for their careful and sophisticated derivations of the attrition rate functions the developers of the Vector-I model are to be commended. Moreover, the assumptions of Vector-I are made at a more detailed level than those of most comparable models. However, more detail in these assumptions makes the global independence assumption less tenable, so the ability of the model to quantitatively represent the effect of small variations in parameter values (or possibly even moderate variations) should be viewed with at least a healthy amount of skepticism. The one assumption of independence is much grosser than many of the other assumptions.

It should also be emphasized that "line of sight" may not be the only reasonable "micro" phenomenon on which a detailed set of assumptions may be based. In particular, there may be certain situations in which targets are sufficiently numerous that loss of a line of sight could be accounted for in kill rates, without the additional mathematics. Of course, this is not an argument against the generality of the model; generality is always desirable, except when it creates false impressions. Similarly, some other physical process might be chosen as basic.

We now proceed to discuss other attrition computations carried out within the model.

Personnel losses in combat at the FEBA are computed from weapon system attrition in the following manner:

$$\Delta p = \sum_i c_i \Delta n_i \ , \tag{3}$$

where

Δp = personnel attrition,

c_i = number of personnel killed when one type i weapon is destroyed.

The c_i are user-specified inputs to the model. Similar calculations apply to personnel losses in the other interactions discussed below.

Attrition to units at the FEBA due to artillery fire is computed in the following manner:

$$\Delta n_i = n_i(1-(1-f)^m) \ , \tag{4}$$

where

m = number of artillery rounds fired,

f = fraction of targets destroyed by one round.

The "kill fraction" f, which may alternatively be interpreted as the probability that a single round kills a particular target, depends on the type of target and type of artillery. Equation (4) is simply a multiple shot binomial equation.

For rear area attrition due to artillery the equation used is

$$\Delta n_i = mk_i \ , \tag{5}$$

where

m = number of rounds,

k_i = number of targets of type i killed per round.

The same equation is also used for counterbattery fire; the k_i are inputs to the model and depend on the type of shooting weapon.

Note that (4) may be approximated by the Lanchester-linear equation

$$\Delta n_i = n_i(fm)$$

and that (5) is of Lanchester-square form. The report [4] justifies this in terms of differing physical situations, especially the deployment of targets.

Air-to-Air Attrition

An attacking air group contains both attack aircraft and escorts and is vulnerable first to the opposition's interceptors. Escorts must attack interceptors on a one-on-one basis. Thus the probability that a particular type i escort engages a particular type j interceptor is

$$q_{ij} = p_{ij} \min\{\frac{1}{E},\frac{1}{I}\} , \tag{6}$$

where

I = total number of interceptors (of all types),

E = total number of escorts,

p_{ij} = probability of engagement given decision to engage.

Here one must interpret

$$\min\{\frac{1}{E},\frac{1}{I}\}$$

as the probability that a particular type i escort decides to engage a particular type j interceptor. The one-on-one engagement hypothesis implies that the number of engagements is at most $\min\{I,E\}$. This must also be the maximum number of decisions to engage and is in fact the actual number of decisions to engage. Note that since it is escorts which seek to engage the interceptors, there is an implicit assumption of perfect coordination and communication among the escorts. If there are $\min\{I,E\}$ decisions to engage and IE (escort, interceptor) pairs, then the probability that a particular escort decides to engage a particular interceptor is

$$\frac{1}{IE} \min\{I,E\} = \min\{\frac{1}{I},\frac{1}{E}\} ,$$

as used in (6). The attrition to type j interceptors is then

$$\Delta I_j = I_j \sum_i E_i k_{ij} q_{ij} \tag{7}$$

and that to type i escorts is

$$\Delta E_i = E_i \sum_j I_j k'_{ji} q_{ij} , \tag{8}$$

where

I_j = number of interceptors of type j,
E_i = number of escorts of type i,

and k_{ij}, k'_{ji} are probabilities of kill given engagement. One must recall that for each particular type i escort and each particular type j interceptor, q_{ij} is to be interpreted as the probability that an engagement occurs involving those two particular aircraft.

A possible alternative to this equation would be the barrier penetration model proposed in [2], which seems to handle one-on-one engagements more sensibly.

The remaining

$$I_j^{(1)} = I_j(1- \sum_i E_i q_{ij})$$

interceptors of type j <u>not engaged</u> by escorts proceed to engage the attack aircraft, again only by means of one-on-one duels. The number of attack aircraft engaged is thus

$$A = \min\{\sum_j I_j^{(1)}, A_0\} ,$$

where

A_0 = total number of attack aircraft (all of which are of one type in any given encounter).

Note the implicit assumption of perfect coordination among defenders. The number of interceptors of type j killed by the attack aircraft is then computed according to the equation

$$\text{(9)} \qquad \Delta I_j^{(1)} = p_j \frac{I_j^{(1)}}{\sum_k I_k^{(1)}} A ,$$

where p_j is a probability of kill given engagement. This is an equation of Lanchester-square form (cf. process S3a of [5]) with engaged attackers allocated proportionally among different types of interceptors. The assumptions implicit in (9) disallow representation of differing engagement capabilities of different types of interceptors. Note that all attack aircraft are engaged provided interceptors outnumber attackers, and vice versa. The term "engaged" is evidently used in a different sense here from that in the escort-interceptor interaction. We also remark that (9) and equation (10) below are obtained by replacing the random numbers of interceptors penetrating the escort screen by their expectations. No explanation is given as to why a square law equation is appropriate here, whereas a linear law equation was appropriate for the interceptor-escort interaction. One possible asymmetry between the two situations is that interceptors and escorts are thought of as engaging in one-on-one duels, whereas several interceptors together engage one attack aircraft. But it is stated above that duels between interceptors and attackers are also of the one-on-one variety. This distinction between several-on-one and one-on-one, it should be noted, accords with a square-law/linear-law distinction made in [7].

The number of attackers killed in interceptors is then computed using the equation

$$(10) \qquad \Delta A = A \frac{\sum_j q_j I_j^{(1)}}{\sum_\ell I_\ell^{(1)}} ,$$

where the q_j are conditional probabilities of kill given engagement. No precise analogue of (10) appears in [5].

Ground-to-Air Attrition

Attrition of aircraft caused by ground-situated air defense sites is calculated by means of a multiple shot binomial attrition equation. There are two types of air defenses, long range and short range; within a given sector there are M_1 long range sites uniformly distributed over the sector, M_{21} short range sites distributed over a forward area near the FEBA and M_{22} short range sites distributed over the rear portion of the sector. The latter differentiation allows for differential effectiveness of short range sites. The number of attacking aircraft killed is then computed as

$$(11) \qquad \Delta A = A[1-(1-p_1)^{M_1}(1-p_{21})^{M_{21}}(1-p_{22})^{M_{22}}] ,$$

where

A = number of attacking aircraft,

and p_1, p_{21}, p_{22} are probabilities of overflight and kill. Thus different sites act independently, and any aircraft is equally likely to be killed by any site of a given type. Moreover, attacks by different sites on a given aircraft are independent, in the sense that whether the aircraft escapes one site is independent of whether any other sites are evaded. Some difficulties with assumptions of this form are discussed in Section 3 of [2].

Escorts of attacking aircraft are treated entirely analogously; we therefore omit a detailed description.

Attacking aircraft are also vulnerable to target area defenses. If the aircraft are not on the air defense suppression mission (which is treated differently, as we discuss below) the number of aircraft killed by target area defenses is computed by the equation

$$(12) \qquad \Delta A^{(1)} = A^{(1)}[1-(1-r_1)^{s_1}(1-r_2)^{s_2}] ,$$

where

$A^{(1)}$ = number of attacking aircraft (those which have survived air defense sites),

s_i = number of type i target defense sites,

r_i = probability an attacking aircraft is killed by a particular site of type i.

The same comments apply to (12) as to (11).

For aircraft whose mission is suppression of long range air defense sites, attrition is computed using the equation

$$(13) \qquad \Delta B = (1-r)^s a(B) + (1-(1-r)^s)^B ,$$

where

B = number of aircraft attacking site,

s = number of short range sites defending the long range site,

r = probability an attacking aircraft is killed by a particular short range site,

$a(B)$ = number of attack aircraft killed when B aircraft attack the long range site.

The function a is a tabular input to the model.

The equation (13) is obtained using the following reasoning. The B attacking aircraft are vulnerable first to the site itself, which destroys $a(B)$ of them. The remaining $B - a(B)$ attack aircraft are then vulnerable to the short range sites defending the long range site; of these aircraft

$$(B-a(B))(1-(1-r)^s)$$

are destroyed. The total number of aircraft destroyed is then

$$\Delta B = a(B) + (B-a(B))(1-(1-r)^s) \ . \tag{14}$$

Simple algebraic manipulations convert (14) to (13).

Outbound attrition to aircraft which have survived target defenses is calculated using equation (11) as described above. For aircraft on CAS missions, air-to-ground damage is assessed before aircraft attrition.

Air-to-Ground Attrition

The probability that a given air defense site is destroyed by aircraft on the air defense suppression mission which attack that site is

$$p = p(B) \ , \tag{15}$$

where

B = number of aircraft attacking that site,

and p is a user-defined function giving the probability that a site is killed as a function of the number of aircraft attacking it. The number of sites destroyed is thus

(16) $$\Delta S = \sum_i p(B_i) ,$$

where B_i is the number of aircraft attacking site i. In addition, each attack on a site leads to the destruction of

(17) $$\Delta t_k = q_k B$$

subsidiary targets of type k, where

q_k = number of type k targets destroyed by one attacking aircraft.

The model report [4] contains no mention of explicit representation of suppression of air defense sites within the Vector-I model (i.e., the possibility that an attack on a site can make it unfunctional for that day without destroying it completely). Some similar models do contain explicit modeling of some suppressive effects; cf. [1,3] for the treatment in IDAGAM I.

With the exceptions noted below, all other aircraft attrition to ground targets is computed using equations of the form (17). We feel there is little justification for such equations and that sensible and practical alternatives are available (e.g., single shot binomial). Depending on the exact form of the q_k, equation (17) can be Lanchester-square in form, Lanchester-linear, or of some entirely different form.

Damage by shallow CAS sorties to weapons of a given type in maneuver units is computed using a mixed-mode Lanchester equation of the following form:

(18) $$\Delta W = cS + W(1-(1-c')^S) ,$$

where

W = number of weapons,

S = number of sorties,

c = number of weapons killed by impact-lethality weapons per sortie,

c' = fraction of weapons destroyed by area fire weapons per sortie.

The parameters c and c' depend on the type of weapon system and the type of attacking aircraft. These calculations are performed separately for each type of weapon, and can therefore lead to overestimates of attrition unless the values of c and c' account for this. The claim that (18) represents a mixed-mode Lanchester equation is based on the approximation

$$\Delta W \sim cS + c'SW,$$

obtained by replacing $(1-c')^S$ by $1 - c'S$. Note, however, that c' is a fraction of weapons killed and can hence be expressed as

$$c' = c''/W \ ,$$

whereas c is a number of weapons killed, and is thus expressible, where c_0 is some constant, as

$$c = c_0 W \ .$$

Upon making these substitutions in (18) one obtains

$$\Delta W = c_0 WS + c''S \ .$$

Here the "linear-law" term arises from impact-lethality weapons and the "square-law" term from area fire weapons, in a manner consistent with [7].

The model computes numbers of sheltered and unsheltered aircraft on the ground and vulnerable to attack under a number of rather reasonable assumptions (e.g., shelters are used as much as possible, all shelters are indistinguishable and equally vulnerable, $\cdots$) as well as some questionable assumptions (unsheltered aircraft are in a distinctly separate area from sheltered aircraft, live targets can be distinguished from dead targets).

The potential number of unsheltered aircraft killed is computed by means of the equation

$$(19) \qquad U^{(0)} = \sum_i [a_i M_i + U(1-(1-f_i)^{M_i})] \ ,$$

where

M_i = number of attacking aircraft of type i,

a = number of unsheltered aircraft destroyed by one attacking aircraft, using direct fire weapons,

f = fraction of unsheltered aircraft destroyed by one attacking aircraft, using area fire weapons.

This potential total is then adjusted to prevent overkill; that such an adjustment is necessary is an admission that the equation is incorrect in at least some cases. The error may, however, be relatively small. This equation also is mixed-mode Lanchester in form.

Computations of the attrition of sheltered aircraft and shelters themselves are entirely analogous. If a shelter is destroyed, its contents necessarily are, but not conversely. The attrition computed by (19) is allocated proportionately among types of target aircraft.

A helicopter effects model is included but is entirely in the form of a tabular input.

General Comments

The following are some general comments concerning attrition methodology in Vector-I:

1. The main attrition equation is an approximation to the stochastic attrition process specified by a consistent, but uneven, set of assumptions, or to the

solution of a deterministic differential equation. It differs from one of the attrition equations available in IDAGAM I [1,3] mainly in terms of the method used to compute attrition coefficients. In this context the specific differences are manner of dependence of attrition coefficients on the entire set of targets and the level of detail of the inputs to this calculation. Whether, in view of the independence assumption and the level of detail in other parts of the model (and in the model as a whole), this represents a significant contribution to combat modeling, is not certain. It does, to our taste, represent a contribution in the sense that the underlying assumptions are known and carefully stated.

2. The role of the independence assumption in (1) may be more crucial than the report [4] leads one to believe. While, as we discussed above, this assumption is probably necessary on grounds of tractability and is at least plausible, it is a much grosser assumption than the others underlying (1). It is possible that the detailed assumptions could be replaced by similarly gross assumptions without significantly altering the capabilities of the model.

3. The use of (1) as an approximation to a stochastic attrition equation (or, more properly, to a computation of expected attrition resulting from a particular stochastic attrition process) or to the solution of a differential equation introduces errors possibly of the same magnitude as the underlying independence assumption. Moreover, the short time periods for which (1) may be valid are those at which the limit arguments used to obtain the attrition coefficients are least valid. This is an additional source of error.

4. Other parts of the model contain assumptions which are comparable in level of detail to the independence assumption. Hence, the model contains several assumptions which are considerably grosser than the detailed assumptions used to compute the attrition coefficients. Because of this, its ability to resolve, except qualitatively, the effects of variations in the "micro" inputs is limited.

5. As is true in any iterative deterministic simulation, random variables in Vector-I are replaced by their expectations (or approximations thereof) for inputs to succeeding calculations (if one adopts the stochastic interpretation of the main attrition equation (1)). Based on experiences with Monte Carlo simulations of homogeneous stochastic Lanchester attrition processes [8], we feel that the error so introduced is no greater than that introduced by the other assumptions, such as those underlying (1). It should be noted, however, that the attrition coefficients calculated from primitive data are themselves expectations, which is further grounds for doubting the usefulness of such detailed inputs.

6. The attrition equation (1) is used in Vector-I to model an assault on a hasty defense, which is the principal, but not the only, ground interaction in the model (among the others are advances and crossings of urban areas and rivers). All the other interactions are modeled by user-input tables. The only justification for this is an argument that the attrition involved is so small that such procedures are acceptable. This assertion is questionable, particularly for protracted, unintense conflicts.

To conclude, Vector-I contains possible contributions to modeling attrition of ground forces in having attrition coefficients dependent on internal factors and a procedure for computing these attrition coefficients from more primitive data (which must be given in range-dependent, movement-dependent,... form). Whether either contribution is significant in a practical sense is, we believe, doubtful in view of the level and number of assumptions necessarily required to obtain a tractable model and in view of the relative lack of attention given to modeling other attrition interactions. In particular, we feel that the model may be incapable of resolving effects of even substantial variations in its "primitive" inputs. The model certainly makes a contribution by having an explicit set of hypotheses from which the main attrition equation can be derived.

6. TACTICAL DECISION RULES

As should be clear from the preceding sections the tactical decision rules play a crucial role in the Vector-I model. The basic purpose of the rules is to set the values of decision variables (i.e., to choose among alternatives) based on the current values of certain of the state variables (i.e., force strengths and positions, reserve levels, supply levels). Any programmable function is acceptable as a tactical decision rule; the rules must incorporate suitable safeguards against patently impossible actions, such as assignment of more forces than exist. The general areas in which the rules function are allocations of resources to sectors, retirement and commitment of maneuver units at the FEBA, allocation of reserve resources, the decision move at the FEBA (either to seek to advance or to disengage and withdraw) and all activity assignments.

Specifically, tactical decision rules are used in the following contexts for each time period:

(1) assignment of newly deployed aircraft to sectors;
(2) assignment of newly deployed helicopters to sectors;
(3) assignment of newly deployed maneuver units, personnel and weapons systems to sectors;
(4) determination of forces to be retired from the FEBA to reserve status;
(5) commitment of reserve units to the FEBA and determination of strength at which they are committed;
(6) assignment of individual replacements to segments;
(7) determination of sector intent variables (e.g., seek to advance, seek to hold a defensive position);
(8) setting of segment plans;

(9) determination of segment activity (one of the types of ground interaction listed in Section 4);

(10) assignment of aircraft to missions;

(11) determination of number and allocation of artillery rounds;

(12) assignment of helicopter sorties to segments;

(13) choice of data base for certain calculations (based on type of infantry, movement, ...);

(14) minefield assignment;

(15) determination of occurrence of attacks on hasty defense, calls for support fire, disengagement (possible results: defender breaks off, defender calls for artillery support or air support, no action; similar choices for attacker); and

(16) FEBA smoothing within sectors and sector-to-sector.

As previously noted, these rules impart great potential flexibility and power to the model, which have probably not yet been fully exploited. They also serve the laudable purposes of collecting in one place a number of related inputs and problems, and of forcing upon the user an awareness of the assumptions underlying this portion of a combat simulation. With Vector-I the user can fairly easily make changes not easily made in other models and can ascertain the effect of changes in behavioral and organizational factors that are possibly more important to the eventual outcome of a combat than things such as force composition and strength (at least over the ranges ordinarily considered). The user who carefully constructs his own tactical decisions rules has a much better understanding of Vector-I than he does of a model to which he supplies only numerical inputs.

Possible disadvantages are that the user may be ill-prepared to construct these rules; it may well be true that determination of a preferred form for each rule is the responsibility of the model-builder. He should at least make recommendations, of which there are none in [4], but which presumably exist. Finally, there is the possibility that these rules have so much influence on the results of the model (particularly if they are ill-chosen) as to render it nearly useless for its stated purposes.

On the whole, however, the idea seems very commendable and, if not abused, both desirable and effective.

7. INPUTS AND OUTPUTS

As inputs Vector-I requires quantitative force performance data, initial resources and time-phased resource arrivals, and the tactical decision rules. The main outputs are daily and cumulative weapon system losses and personnel casualties, classified by type and by cause; supply levels and consumption, both daily and cumulative, and numbers and locations of currently surviving resources, including reserves. A wide range of secondary outputs is also available. In terms of input and output capabilities, Vector-I does not appear to differ significantly from comparable models such as IDAGAM I [1,3]. The number of inputs required is probably greater because of the use of detailed inputs in computation of the attrition rate functions, but the relative difference in number of inputs is probably not large.

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